**KEITH DEVLIN: Introduction to Mathematical Thinking ASSIGNMENT 2 (for Lecture 2)**

**Kgotso Koete answer sheet, 22 April 2017**

1. Simplify the following symbolic statements as much as you can, leaving your answer in the standard

Symbolic form. (In case you are not familiar with the notation, I’ll answer the first one for you.)

(a) (π > 0) ∧ (π < 10) [Answer: 0 < π < 10.]

(b) (p ≥ 7) ∧ (p < 12) [Answer: 7 ≤ p < 12.]

(c) (x > 5) ∧ (x < 7) [Answer: 5 < p < 7.]

(d) (x < 4) ∧ (x < 6) [Answer: 4 < x < 6.]

(e) (y < 4) ∧ (y2 < 9) [Answer: (y < 4) ∧ (y2 < 9).]

(f) (x ≥ 0) ∧ (x ≤ 0) [Answer: x = 0]

2. Express each of your simplified statements from question 1 in natural English.

ANSWER:

a) Pie is larger than 0 and smaller than 10.

b) p is larger or equal to 7, and smaller than 12.

c) x is larger than 5 and smaller than 7.

d) x is larger than 4 and smaller than 6.

e) ??

f) x is equal to 0.

3. What strategy would you adopt to show that the conjunction φ1 ∧ φ2 ∧ . . . ∧ φn is true?

ANSWER: If all values from φ1 to φn are true, then the conjunction is also true.

4. What strategy would you adopt to show that the conjunction φ1 ∧ φ2 ∧ . . . ∧ φn is false?

ANSWER: If any value from φ1 to φn is false and at least 1 is true, then the conjunction is also false.

5. Simplify the following symbolic statements as much as you can, leaving your answer in a standard

symbolic form (assuming you are familiar with the notation):

(a) (π > 3) ∨ (π > 10) [Answer: (π > 3) ∨ (π > 10)]

(b) (x < 0) ∨ (x > 0) [Answer: (x < 0) ∨ (x > 0)]

(c) (x = 0) ∨ (x > 0) [Answer: x ≥ 0.]

(d) (x > 0) ∨ (x ≥ 0) [Answer: x ≥ 0.]

(e) (x > 3) ∨ (x2 > 9) [Answer: (x > 3) ∨ (x2 > 9).]

6. Express each of your simplified statements from question 5 in natural English.

ANSWER

a) pi is larger than 3 or pi is larger than 10.

b) x is a negative or a positive number.

c) x is a bigger or equals to 0.

d) x is a bigger or equals to 0.

e) x is larger than 3 or the square of x is larger than 9.

7. What strategy would you adopt to show that the disjunction φ1 ∨ φ2 ∨ . . . ∨ φn is true?

ANSWER: If any value between φ1 and φn is false and at least one is true, then the disjunction is true.

8. What strategy would you adopt to show that the disjunction φ1 ∨ φ2 ∨ . . . ∨ φn is false?

ANSWER: If all values between φ1 and φn are false, then the disjunction is false.

9. Simplify the following symbolic statements as much as you can, leaving your answer in a standard

symbolic form (assuming you are familiar with the notation):

(a) ¬(π > 3.2) [ANSWER: π ≤ 3.2 ]

(b) ¬(x < 0) [ANSWER: x ≥ 0 ]

(c) ¬(x2 > 0) [ANSWER: x = 0]

(d) ¬(x = 1) [ANSWER: 1 < x < 1]

(e) ¬¬ψ [ANSWER: ψ ]

10. Express each of your simplified statements from question 9 in natural English.

ANSWER:

a) pi is smaller or equal to 3.2.

b) x is bigger or equal to 0.

c) x is equal to zero.

d) x is larger or smaller than 1.

e) Psi is true.

11. Let D be the statement “The dollar is strong”, Y the statement “The Yuan is strong” and T

the statement “New US–China trade agreement signed”. Express the main content of each of the

following (fictitious) newspaper headlines in logical notation. (Note that logical notation captures

truth, but not the many nuances and inferences of natural language.) How would you justify and

defend your answers?

(a) Dollar and Yuan both strong

ANSWER: D = True ^ Y = True (True meaning strong/favorable). What makes the statement valid is the word ‘both’ in the headline.

(b) Yuan weak despite new trade agreement, but Dollar remains strong

ANSWER: (Y = False V T = True) ^ (D = True V T = True).

(c) Dollar and Yuan can’t both be strong at same time.

ANSWER: D = True ^ Y = False V D = False ^ Y = True. It’s with the Dollar is strong and the Yen is weak or the other way around.

(d) New trade agreement does not prevent fall in Dollar and Yuan

ANSWER: T = False V (D = True ^ Y = True). Trade is favorable or (Dollar is unfavorable and Yen is unfavorable).

(e) US–China trade agreement fails but both currencies remain strong

ANSWER: T = False V (D = True ^ Y = True). Trade is unfavorable or (Dollar is favorable and Yen is favorable).

TWO TO THINK ABOUT AND DISCUSS WITH OTHER STUDENTS

1. In US law, a trial verdict of “Not guilty” is given when the prosecution fails to prove guilt. This, of

course, does not mean the defendant is, as a matter of actual fact, innocent. Is this state of affairs

captured accurately when we use “not” in the mathematical sense? (i.e., Do “Not guilty” and “¬

guilty” mean the same?) What if we change the question to ask if “Not proven” and “¬ proven”

mean the same?

ANSWER: “Not guilty” and “¬guilty” do not have the same meaning because the negation of “¬guilty” is ‘innocent’. This of course is not what the legal system means by “not guilty”. In contrast “Not proven” and “¬ proven” have the same meaning both in court and mathematics because they have the same negation being “proven”.

2. The truth table for ¬¬φ is clearly the same as that for φ itself, so the two expressions make identical

truth assertions. This is not necessarily true for negation in everyday life. For example, you might

find yourself saying “I was not displeased with the movie.” In terms of formal negation, this has

the form ¬(¬ pleased), but your statement clearly does not mean that you were pleased with the

movie. Indeed, it means something considerably less positive. How would you capture this kind of

use of language in the formal framework we have been looking at?

ANSWER: ¬(¬ pleased) cannot be used to express the satisfaction level of the movie viewer. A more accurate description of the satisfaction levels in mathematical terms would be not pleased < satisfaction level < pleased.